

# Comparison of Two Solutions for the NuMI Beamline

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## Abstract

Two beamline designs are compared. The quantities of interest are acceptance, sensitivity to extraction error, distance to aperture, and beam loss. Various beam scenarios are investigated.

## 1 Description of the Beamlines

This note compares two beamlines: a “Tweaked Baseline” and a proposed “FODO” design. The “Tweaked Baseline” is an optimization of the baseline NuMI beamline. It uses a minimum number of quadrupoles to transport the beam from MI extraction to the NuMI target; it has no quadrupoles in the carrier tunnel. The “FODO” uses a FODO structure as wherever possible, pushing this structure into the allowed region of the carrier tunnel. Table 1 summarizes the models.

## 2 Extraction Error Study

To study the sensitivity of the beamline to extraction errors, an initial (physical) phase space of 1 mm by  $1\mu\text{radian}$  (horizontal and vertical), and  $\Delta p/p = \pm 1.0 \times 10^{-4}$  was transmitted through the beamline<sup>1</sup>. Thus, the centroid of any beam falling within the specified phase-space and momentum bite will remain within the envelope shown. Figures 1 and 2 show the results.

## 3 Acceptance

The acceptance of the beamline is determined using the ray-tracing program TURTLE<sup>2</sup>. Progressively larger phase space annuli are traced throughout the beamline until a ray intersects an aperture. The largest annulus which does not hit an aperture is the acceptance.

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<sup>1</sup>See D.C. Carey, et al, “Third-Order TRANSPORT with MAD Input”, Fermilab-Pub-98-310, for computational details.

<sup>2</sup>D.C. Carey, et al, “TURTLE with MAD Input”, Fermilab-Pub-99/232

### 3.1 Simple Calculation

Initially, annuli are thrown in horizontal phase space, and the vertical phase space is set to zero. Then, vertical annuli are thrown, and the horizontal phase space is set to zero. For all runs,  $\Delta p/p = 0$ . The results are shown in Table 2.

### 3.2 Momentum Bite Included

Next, the admittance is calculated for a non-zero momentum bite. Also, both horizontal and vertical phase space are populated at the same time (though they are uncorrelated). Specifically, each ray is generated to lie on an annulus in horizontal phase space, and on another annulus in vertical phase space. This ray is then given a momentum offset of  $\delta p/p = +1.1 \times 10^{-3}$ . The ray is copied, and given a momentum offset of  $\delta p/p = -1.1 \times 10^{-3}$ . Finally, a grid search is done in vertical and horizontal admittance to maximize the product. Table 3 summarizes the results. Because the emittance of the Main Injector is limited to about  $500 \pi \cdot mm \cdot mradian$ , “FODO” is shown with the admittance truncated to  $500 \pi \cdot mm \cdot mradian$ .

## 4 Aperture Plots

Figures 3 and 4 show the aperture figure-of-merit for both beamlines. To calculate the FOM, one calculates the beam envelope for a given fraction of the beam (including momentum spread) and takes the distance to each aperture; the result is then divided by the  $1\sigma$  beam size (excluding momentum spread). So, for a given fraction of the beam,  $x$ , the fom for  $x$  is defined as:

$$fom(x) = \frac{aperture - \sqrt{\epsilon(x)\beta + \eta^2(\Delta(x)p/p)^2}}{\epsilon_{1\sigma}}$$

For this calculation, we assume a  $40\pi(95)$  normalized emittance and  $\sigma_p/p = 7.0 \times 10^{-4}$ .

Figures 5 and 6 show the 99.99% contour of a  $40\pi$ ,  $\sigma_p/p = 7.0 \times 10^{-4}$  beam superimposed on the beamline apertures.

## 5 Momentum Acceptance

In this study, the vertical and horizontal emittances were held fixed at  $40\pi$ , and the momentum bite was increased. As in the previous acceptance study, TURTLE was used to trace rays. Table 4 summarizes the results.

## 6 Beam Loss

This study investigated beam loss for various combinations of emittance and momentum bite. For each study, 2 dimensional distribution, gaussian in phase

space, was generated. Then, the inner 95% of the distribution was removed (leaving only the outer 5%). Enough particles were generated so that 1,000,000 particles were in the remaining outer 5%. The distribution in momentum was gaussian—nothing was deleted. TURTLE was then used to track the particles through the beamlines and identify any losses. Both beamlines used the same distribution.

Figures 7 and 8 show representative loss plots for the two models. Table 5 shows the amount of beam lost, and Table 6 shows a figure of merit, for various scenarios in each beamline. The 95% emittance and the  $\sigma_p/p$  are listed for each scenario. The figure of merit is found by taking the ratio of each loss to the allowed loss, and then finding the maximum.

Model	Uses FODO	Has quadrupoles in carrier pipe
Tweaked Baseline	No	No
FODO	Yes	Yes

Table 1: Summary of the key features of the beamline models compared in this note.

Model	Horizontal [ $\pi \cdot mm \cdot mr$ ]	Vertical [ $\pi \cdot mm \cdot mr$ ]
Tweaked Baseline	110	444
FODO	1116	1321

Table 2: Normalized admittance, in  $\pi \cdot mm \cdot mr$ , of each model. Admittance of each plane is calculated independently.

Model	Horizontal [ $\pi \cdot mm \cdot mr$ ]	Vertical [ $\pi \cdot mm \cdot mr$ ]	Aperture [ $mm^2$ ]
Tweaked Baseline	35	200	7000
FODO	648	707	458136
FODO	500	500	250000

Table 3: Normalized admittance, including  $\delta p/p = \pm 1.1 \times 10^{-3}$ , for each model. Horizontal and vertical phase space are populated simultaneously, though uncorrelated. “FODO” is shown a second time with the admittance truncated to the emittance of the Main Injector.

Model	$\pm \delta p/p$
Tweaked Baseline	$2.3 \times 10^{-3}$
FODO	$3.8 \times 10^{-3}$

Table 4: Maximum momentum acceptance at  $40\pi$ .

Model	$25\pi$ $3.5 \times 10^{-4}$	$40\pi$ $3.5 \times 10^{-4}$	$40\pi$ $7.0 \times 10^{-4}$	$40\pi$ $14 \times 10^{-4}$
Tweaked Baseline	1.8%	2.7%	2.9%	3.2%
FODO	0	0	$1.2 \times 10^{-6}$	$6.5 \times 10^{-4}$

Table 5: Beam loss for various scenarios. Losses of less than 1% are listed as a fraction.

Model	$25\pi$ $3.5 \times 10^{-4}$	$40\pi$ $3.5 \times 10^{-4}$	$40\pi$ $7.0 \times 10^{-4}$	$40\pi$ $14 \times 10^{-4}$
Tweaked Baseline	115	238	233	216
FODO	0	0	$1.2 \times 10^{-2}$	6.5

Table 6: Beam loss figure-of-merit for various scenarios. The fom is the maximum ratio between projected loss and allowed loss.

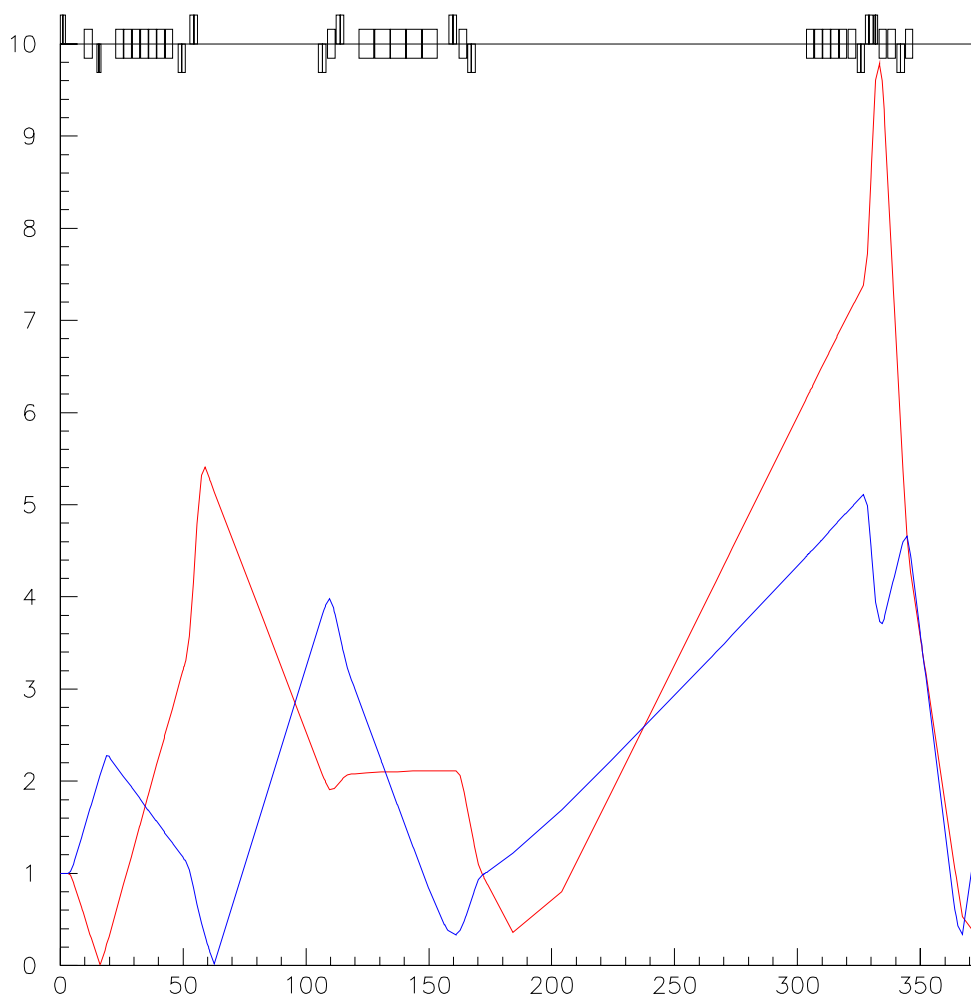


Figure 1: “Tweaked Baseline”. The horizontal envelope is blue.

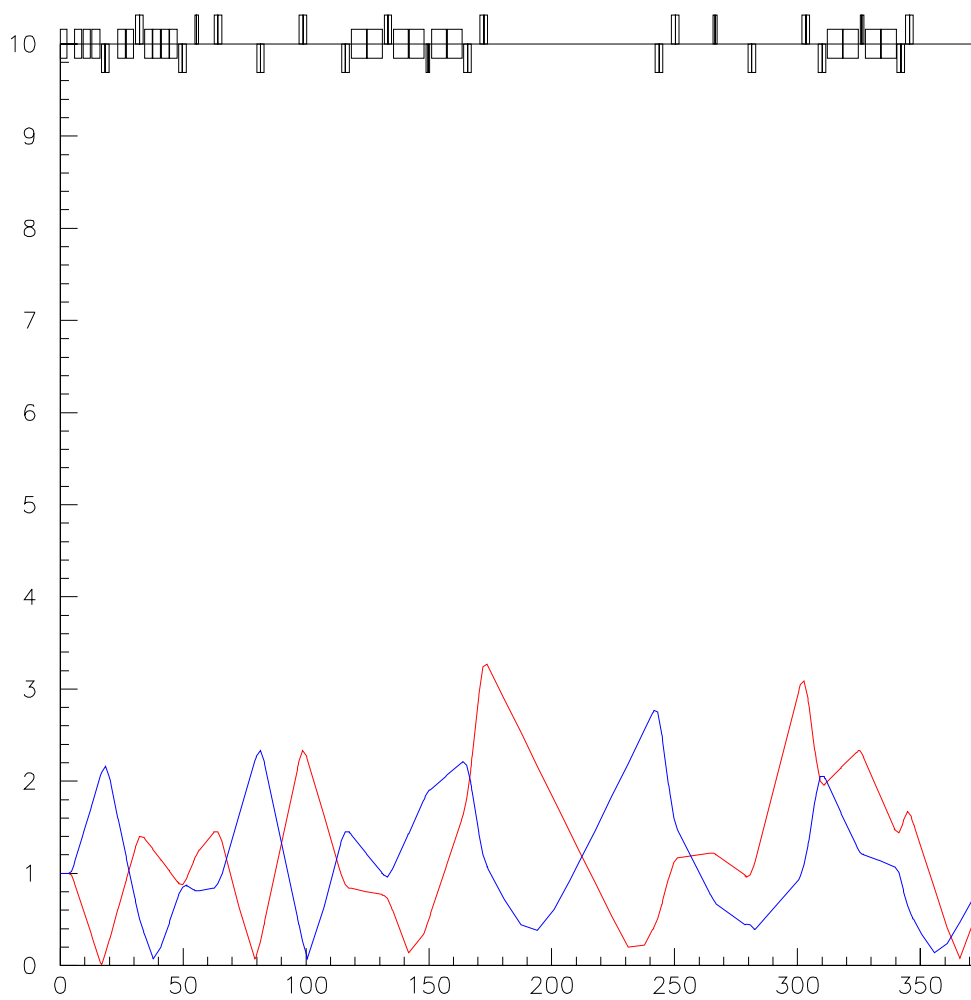


Figure 2: “FODO” The horizontal envelope is blue.

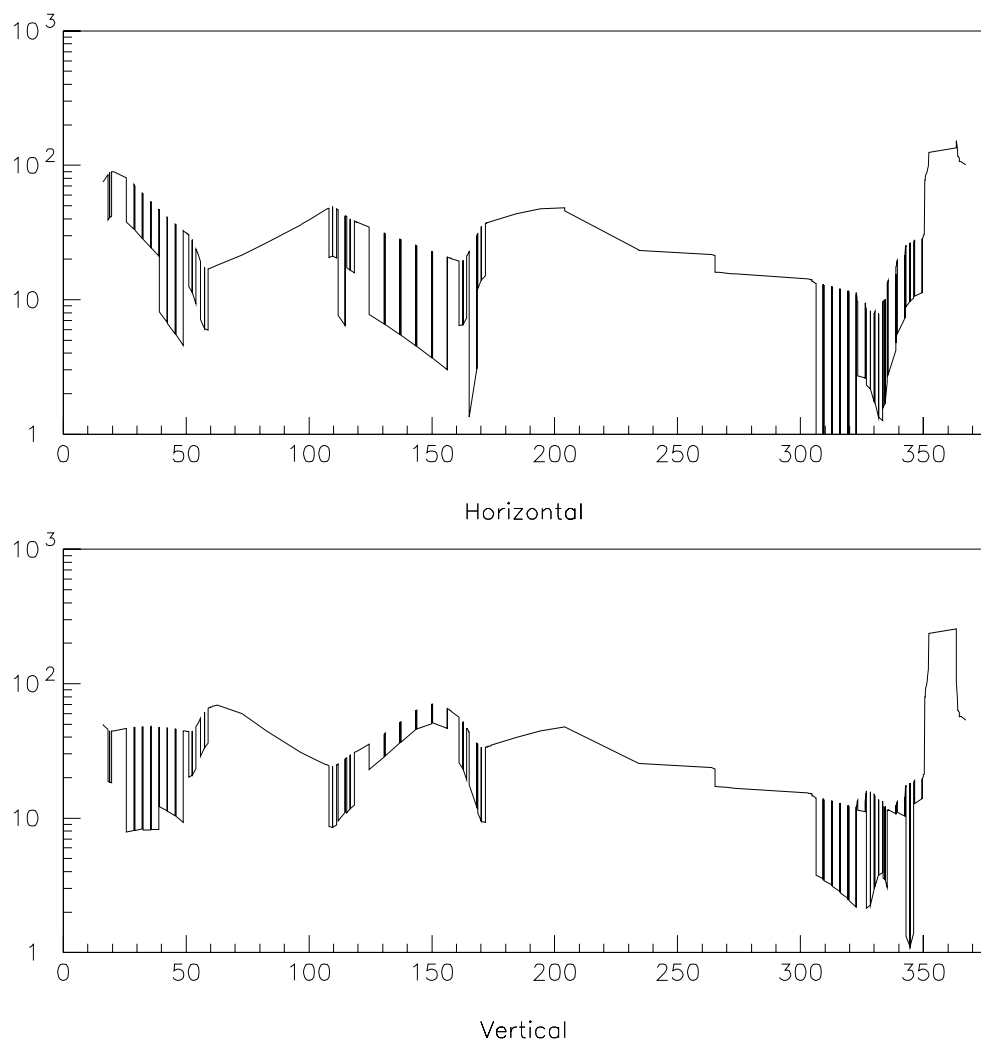


Figure 3: "Tweaked Baseline". CDMFOM for the 99.99% envelope.

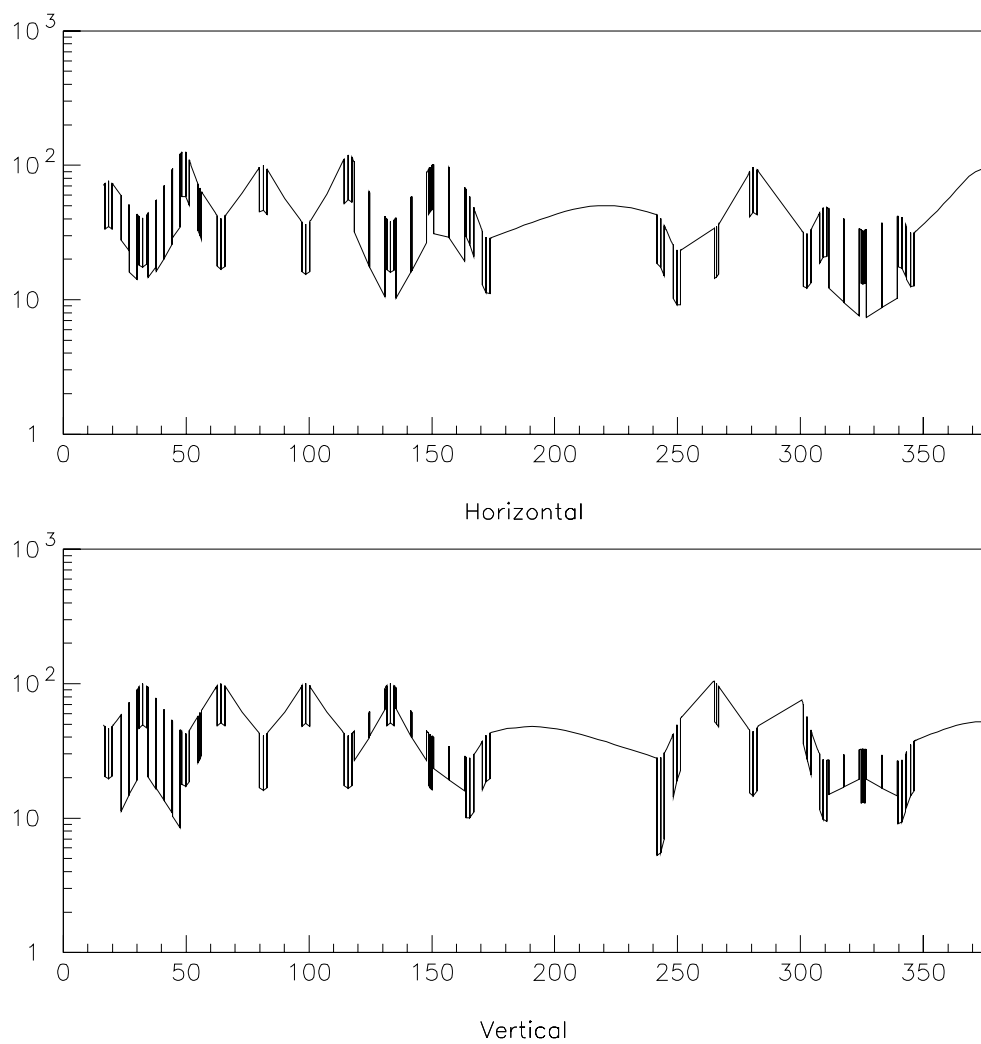


Figure 4: “FODO”. CDMFOM for the 99.99% envelope.



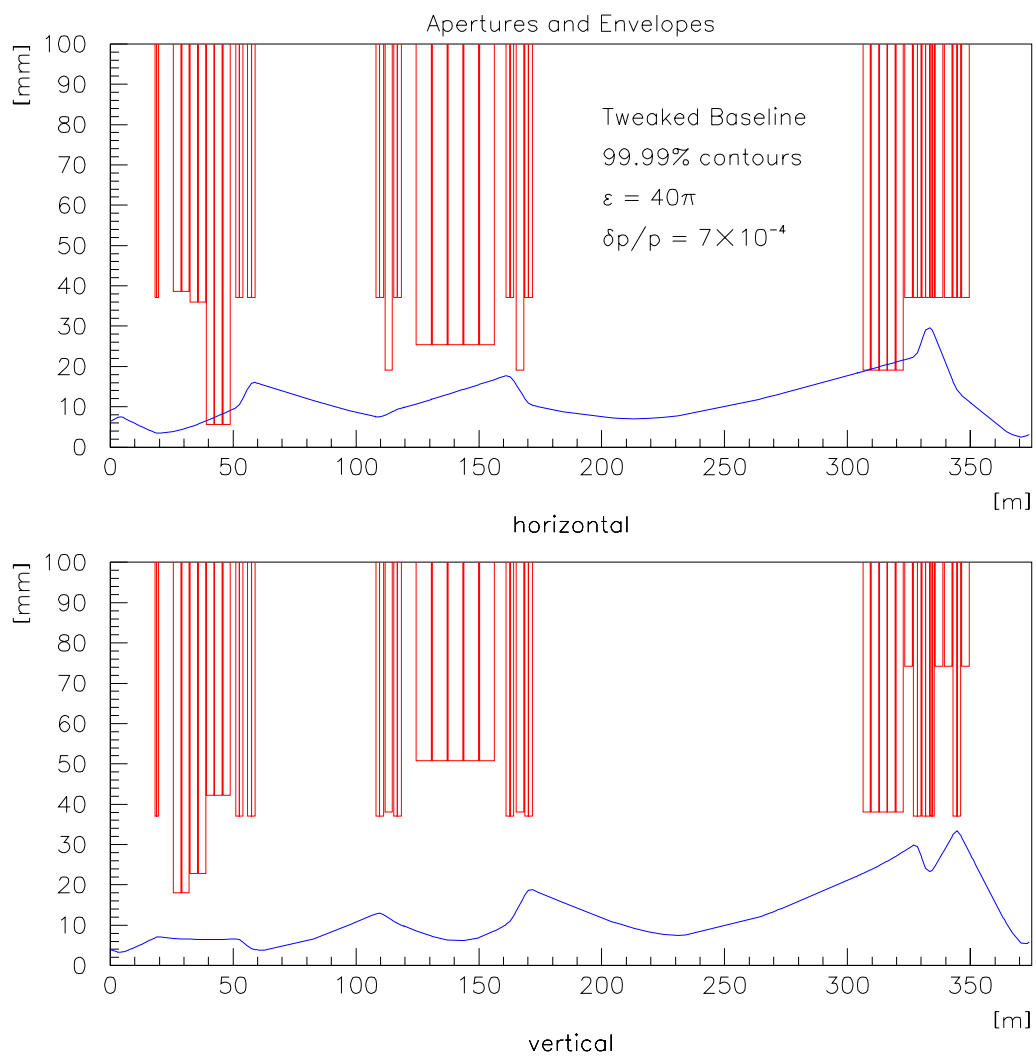


Figure 5: “Tweaked Baseline”. 99.99% envelope superimposed on apertures.

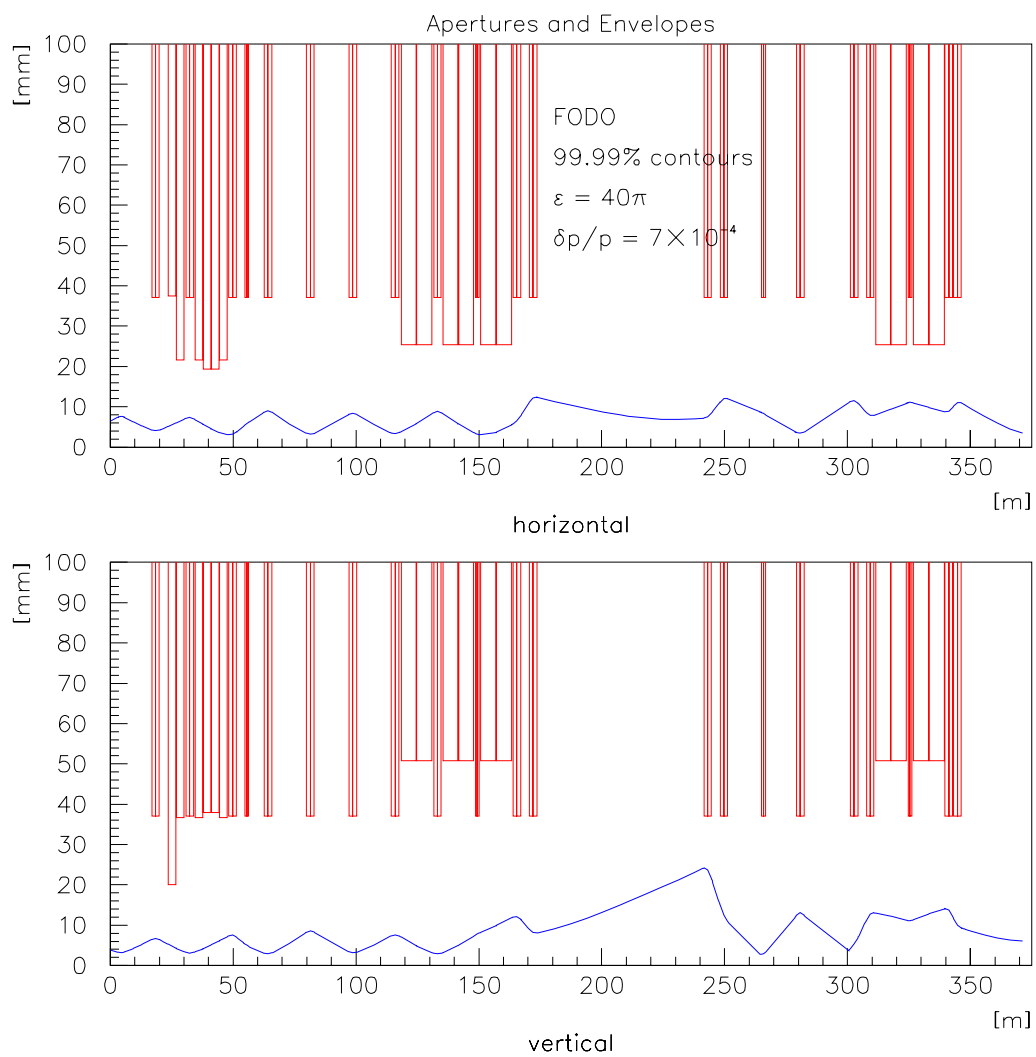


Figure 6: “FODO”. 99.99% envelope superimposed on apertures.

Figure 7: “Tweaked Baseline”. Loss points.

Figure 8: “FODO”. Loss points.